Problem Set 8: Due Monday, February 24

Problem 1: Consider the unique linear transformation $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ such that

$$T\left(\begin{pmatrix} 9 \\ 4 \end{pmatrix}\right) = \begin{pmatrix} 1 \\ -5 \end{pmatrix} \quad \text{and} \quad T\left(\begin{pmatrix} 2 \\ 1 \end{pmatrix}\right) = \begin{pmatrix} -2 \\ 3 \end{pmatrix}.$$

Determine, with explanation, the value of

$$T\left(\begin{pmatrix} 6 \\ 2 \end{pmatrix}\right).$$

Problem 2: Compute

$$\begin{pmatrix} 4 & 3 \\ -7 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 5 \end{pmatrix}.$$

Describe what your computation means in terms of a linear transformation. Use Problem 1 above as a guide.

Problem 3: Define $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ by letting $T(\vec{v})$ be the point on the line $y = x + 1$ that is closest to $\vec{v}$. Is T is a linear transformation? Explain.

Problem 4: Consider the unique linear transformation $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ such that

$$T\left(\begin{pmatrix} 1 \\ -1 \end{pmatrix}\right) = \begin{pmatrix} 1 \\ 4 \end{pmatrix} \quad \text{and} \quad T\left(\begin{pmatrix} -2 \\ 3 \end{pmatrix}\right) = \begin{pmatrix} 2 \\ 7 \end{pmatrix}.$$

What is $[T]$? Explain.

Problem 5: Suppose that $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ and $S\colon \mathbb{R}^2 \to \mathbb{R}^2$ are both linear transformations. Show that $T \circ S\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation.

Problem 6: For each of following, consider the linear transformation $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ that has the given matrix as its standard matrix. Describe the action of T geometrically. It may help to plug in a few points and/or make some case distinctions.

a. $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

b. $\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$ for a fixed $k \in \mathbb{R}$ with $k > 0$.

c. $\begin{pmatrix} k & 0 \\ 0 & 1 \end{pmatrix}$ for a fixed $k \in \mathbb{R}$ with $k > 0$.