Problem Set 21: Due Friday, May 1

Problem 1: Consider the unique linear transformation $T\colon \mathbb{R}^4 \to \mathbb{R}^3$ with

$$[T] = \begin{pmatrix} 1 & 0 & 2 & -1 \\ 3 & 1 & 9 & -5 \\ -1 & 2 & 4 & -2 \end{pmatrix}.$$

a. Find (with explanation) bases for each of range(T) and Null(T).
b. Calculate rank(T) and nullity(T).

Problem 2: Consider the unique linear transformation $T\colon \mathbb{R}^4 \to \mathbb{R}$ with

$$[T] = \begin{pmatrix} 0 & 1 & -3 & 7 \end{pmatrix}.$$

a. Find (with explanation) bases for each of range(T) and Null(T).
b. Calculate rank(T) and nullity(T).

Problem 3: Define $T\colon \mathcal{P}_5 \to \mathcal{P}_5$ by letting $T(f) = f''$, i.e. $T(f)$ is the second derivative of f. Determine, with explanation, both rank(T) and nullity(T).

Problem 4: Let V be the vector space of all 2×2 matrices. Explain why there is no surjective linear transformation $T\colon V \to \mathcal{P}_4$.

Problem 5: Consider the matrix

$$A = \begin{pmatrix} 3 & 2 \\ 5 & 3 \end{pmatrix}.$$

We know from Proposition 2.7.9 that A is invertible, and we also know a formula for the inverse. Now compute A^{-1} using our new method by applying elementary row operations to the matrix

$$\begin{pmatrix} 3 & 2 & 1 & 0 \\ 5 & 3 & 0 & 1 \end{pmatrix}.$$