# Problem Set 19: Due Friday, April 24

**Problem 1:** Working in $\mathbb{R}^4$, find the dimension of the subspace

$$W = \text{Span}\left(\begin{pmatrix} 1 \\ 5 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ -6 \\ 1 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ 8 \\ 2 \\ 2 \end{pmatrix}\right).$$

**Problem 2:** Working in the vector space $\mathcal{F}$ of all functions $f\colon \mathbb{R} \to \mathbb{R}$, define the following:

- $f_1(x) = 2^x$.
- $f_2(x) = 3^x$.

Find, with explanation, the dimension of the subspace $W = \text{Span}(f_1, f_2)$.

**Problem 3:** Let $V$ be the vector space of all $2 \times 2$ matrices. Let

$$W = \left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in V : 2a - c = 0 \text{ and } b + c - d = 0\right\}.$$

It turns out that $W$ is a subspace for $V$ (no need to show this). Find a basis for $W$, and determine $\dim(W)$.
*Hint:* First try to write $W$ as the span of some elements of $V$ by solving the system of equations.

**Problem 4:** Define $T\colon \mathcal{P}_1 \to \mathbb{R}^2$ by letting

$$T(a + bx) = \begin{pmatrix} a - b \\ b \end{pmatrix}.$$

Show that $T$ is a linear transformation.

**Problem 5:** Let $T\colon \mathbb{R}^3 \to \mathbb{R}^3$ be the function

$$T\left(\begin{pmatrix} x \\ y \\ z \end{pmatrix}\right) = \begin{pmatrix} x - y \\ x + z \\ y + z \end{pmatrix}.$$

a. Explain why $T$ is a linear transformation.
b. Give an example of a nonzero $\vec{v} \in \mathbb{R}^3$ such that $T(\vec{v}) = \vec{0}$.
c. Show that $T$ is not injective.