Problem Set 15: Due Monday, April 6

Problem 1: Let V be a vector space, and let W be a subspace of V. Recall that

$$V \setminus W = \{\vec{v} \in V : \vec{v} \notin W\},$$

i.e. $V \setminus W$ is the set of elements of V that are *not* in W. Is $V \setminus W$ always a subspace of V? Sometimes a subspace of V? Never a subspace of V? Explain.

Problem 2: Use Gaussian Elimination to solve the following system:

$$
\begin{array}{rcrcrcl}
x & & & - & z & = & 0 \\
3x & + & y & & & = & 1 \\
-x & + & y & + & z & = & 4.
\end{array}
$$

Problem 3: Find the coefficients $a, b, c \in \mathbb{R}$ so that the graph of $f(x) = ax^2 + bx + c$ passes through the points $(1, 2)$, $(-1, 6)$, and $(2, 3)$.

Problem 4: Is

$$
\begin{pmatrix} 20 \\ 0 \\ 5 \\ 10 \end{pmatrix} \in \mathrm{Span}\left(\begin{pmatrix} 0 \\ 2 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 4 \\ -2 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \\ -1 \end{pmatrix} \right)?
$$

Explain.

Problem 5: Give a parametric description of the solution set of the following system:

$$
\begin{array}{rcrcrcrcl}
x & + & 2y & - & z & & & = & 3 \\
2x & + & y & & & + & w & = & 4 \\
x & - & y & + & z & + & w & = & 1.
\end{array}
$$

Problem 6: Solve the following three systems simultaneously:

$$
\begin{array}{rcrcl}
x & + & 3y & = & 8 \\
2x & + & 5y & = & 15,
\end{array}
\qquad
\begin{array}{rcrcl}
x & + & 3y & = & 3 \\
2x & + & 5y & = & 4,
\end{array}
\quad \text{and} \quad
\begin{array}{rcrcl}
x & + & 3y & = & -10 \\
2x & + & 5y & = & -14.
\end{array}
$$

Notice that the coefficients of x and y are the same, so you should code these simultaneously as one matrix, and go to reduced echelon form (see p. 181-182 of the course notes).