

## Problem Set 11: Due Friday, March 6

**Problem 1:** Let

$$\alpha = \left( \begin{pmatrix} -1 \\ 2 \end{pmatrix}, \begin{pmatrix} 5 \\ 1 \end{pmatrix} \right).$$

a. Show that  $\alpha$  is a basis for  $\mathbb{R}^2$ .

b. Compute  $\left[ \begin{pmatrix} 5 \\ 1 \end{pmatrix} \right]_{\alpha}$ .

c. Compute  $\left[ \begin{pmatrix} 8 \\ 17 \end{pmatrix} \right]_{\alpha}$ .

In each part, briefly explain how you carried out your computation.

**Problem 2:** Consider the unique linear transformation  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  with

$$[T] = \begin{pmatrix} 6 & -7 \\ 4 & -5 \end{pmatrix}.$$

Let  $\alpha = (\vec{u}_1, \vec{u}_2)$  where

$$\vec{u}_1 = \begin{pmatrix} 5 \\ 3 \end{pmatrix} \quad \text{and} \quad \vec{u}_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}.$$

In this problem, we compute  $[T]_{\alpha}$  directly from the definition.

a. Show that  $\alpha = (\vec{u}_1, \vec{u}_2)$  is a basis of  $\mathbb{R}^2$ .

b. Determine  $T(\vec{u}_1)$  and then use this to compute  $[T(\vec{u}_1)]_{\alpha}$ .

c. Determine  $T(\vec{u}_2)$  and then use this to compute  $[T(\vec{u}_2)]_{\alpha}$ .

d. Using parts (b) and (c), determine  $[T]_{\alpha}$ .

**Problem 3:** With the same setup as Problem 2, compute  $[T]_{\alpha}$  using Proposition 3.2.6.

**Problem 4:** Again, use the same setup as in Problem 2. Let

$$\vec{v} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}.$$

In this problem, we compute  $[T(\vec{v})]_{\alpha}$  in two different ways.

a. First determine  $T(\vec{v})$ , and then use this to compute  $[T(\vec{v})]_{\alpha}$ .

b. First determine  $[\vec{v}]_{\alpha}$ , and then multiply the result by your matrix  $[T]_{\alpha}$  to compute  $[T(\vec{v})]_{\alpha}$ .

**Problem 5:** Consider the unique linear transformation  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  with

$$[T] = \begin{pmatrix} 3 & 2 \\ 4 & -1 \end{pmatrix}.$$

Let  $\alpha = (\vec{u}_1, \vec{u}_2)$  where

$$\vec{u}_1 = \begin{pmatrix} -4 \\ -2 \end{pmatrix} \quad \text{and} \quad \vec{u}_2 = \begin{pmatrix} 9 \\ 4 \end{pmatrix}.$$

Compute  $[T]_{\alpha}$  using any method.

**Problem 6:** Let  $A$  and  $B$  be  $2 \times 2$  matrices. Assume that  $A\vec{v} = B\vec{v}$  for all  $\vec{v} \in \mathbb{R}^2$ . Show that  $A = B$ .