Problem Set 1: Due Monday, January 27

Problem 1: For each part, explain your reasoning using a sentence or two.
a. Consider the line in the plane described by the equation $3x - 2y = 12$. Find an example of $a, b, c, d \in \mathbb{R}$ such that

$$x = a + bt$$
$$y = c + dt$$

is a parametric equation for the line.
b. Find two other choices for $a, b, c, d \in \mathbb{R}$ that work for part (a).
c. Consider the line in the plane described parametrically by

$$x = 2 - 3t$$
$$y = 1 + 5t.$$

Find an example of $a, b, c \in \mathbb{R}$ such that the line is described by the equation $ax + by = c$.
d. Find another example of $a, b, c \in \mathbb{R}$ that works for part (c).

Problem 2: Let P be the plane in $\mathbb{R}^3$ that contains the origin and that is parallel to each of the following two vectors:

$$\vec{u} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} \qquad \text{and} \qquad \vec{w} = \begin{pmatrix} -7 \\ 1 \\ 4 \end{pmatrix}$$

In the notes, we discussed one way to parametrize P, and we will discuss this in more detail later. Now find an equation of the form $ax + by + cz = d$ for P. Explain your process using a sentence of two.

Problem 3: Let L be the line in $\mathbb{R}^3$ that is the intersection of the two planes $3x + 4y - z = 2$ and $x - 2y + z = 4$.
a. Using the equations of the planes, determine if the points $(1, 0, 1)$ and $(1, 1, 5)$ are on L.
b. Find a parametric description of L. Explain your process using a sentence or two.
c. Use the parametric description of L to determine if $(5, 2, 3)$ is a point on L. Explain.
Note: Given a point, it seems easier to determine if it is on L using the equations of the planes rather than the parametric description. In contrast, if you want to *generate* points on L, it is easier to use the parametric description (just plug in values for the parameter) than the plane equations.

Problem 4:
a. Do the planes with equations $2x - 3y + z = 7$ and $-4x + 9y - 2z = 3$ intersect? Explain your reasoning.
b. Do the lines described by the two parametric equations

$$\begin{aligned} x &= -4 + 6t \\ y &= 2 + t \\ z &= 1 + 3t \end{aligned} \qquad\qquad \begin{aligned} x &= 4 + 4t \\ y &= 5 - t \\ z &= 9 - 2t \end{aligned}$$

intersect? Explain your reasoning.