Writing Assignment 9: Due Wednesday, April 24

Problem 1: Let V be a vector space, and let U and W be subspaces of V. Recall from Writing Assignment 8 that

$$U + W = \{\vec{v} \in V : \text{There exists } \vec{u} \in U \text{ and } \vec{w} \in W \text{ with } \vec{v} = \vec{u} + \vec{w}\}.$$

Assume that $(\vec{u}_1, \ldots, \vec{u}_m)$ is a basis of U and that $(\vec{w}_1, \ldots, \vec{w}_n)$ is a basis of W.
a. Show that $U + W = \text{Span}(\vec{u}_1, \ldots, \vec{u}_m, \vec{w}_1, \ldots, \vec{w}_n)$.
b. Show that $\dim(U + W) \leq \dim(U) + \dim(W)$.

Problem 2: Let V be a vector space, and let $\vec{u}, \vec{w} \in V$. Show that $(\vec{u}, \vec{w})$ is linearly dependent if and only if either $\vec{u} = \vec{0}$ or $\vec{w} \in \text{Span}(\vec{u})$.