Writing Assignment 8: Due Wednesday, April 17

Problem 1: Let V be a vector space. Suppose that U and W are both subspaces of V. You showed in Writing Assignment 7 that $U \cup W$ might not be a subspace of V. Instead, let

$$U + W = \{\vec{v} \in V : \text{There exists } \vec{u} \in U \text{ and } \vec{w} \in W \text{ with } \vec{v} = \vec{u} + \vec{w}\}.$$

That is, $U + W$ is the set of all vectors in V that can be written as the sum of an element of U and an element of W. Show that $U + W$ is a subspace of V.

Problem 2: Recall that $\mathcal{P}_2$ is the vector space of all polynomial functions of degree at most 2. Let

$$W = \{f \in \mathcal{P}_2 : f(1) = 0\},$$

i.e. W is the set of all polynomial functions f of degree at most 2 such that 1 is a root of f.
a. Show that W is a subspace of $\mathcal{P}_2$.
b. Give an example, with justification, of $f_1, f_2 \in \mathcal{P}_2$ such that $W = \text{Span}(f_1, f_2)$.