Writing Assignment 1: Due Friday, February 1

Problem 1: Determine whether the following statements are true or false. In all cases, explain your reasoning thoroughly in complete sentences.
a. There exists $m, n \in \mathbb{Z}$ such that $34m + 30n = 2$.
b. There exists $m, n \in \mathbb{N}$ such that $5m + 9n = 16$.
c. For all $a \in \mathbb{R}$, we have $a^6 - 4a^3 + 9 \geq 3$.

Problem 2: Define a function $f\colon \mathbb{R}^2 \to \mathbb{R}^2$ by letting

$$f\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) = \begin{pmatrix} -y \\ x \end{pmatrix}.$$

Think of f as transforming the plane as we discussed in class, and as illustrated on p. 9 of the course notes.
a. Plug in a few (at least four) vectors to get some intuition about how f behaves.
b. Using your example input-output pairs from part (a), make a conjecture about how f transforms the plane.
c. Verify as much of your conjecture as possible. For example, if your conjecture is that f scales every vector by a factor of 3, then you should show that

$$f\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) = 3 \cdot \begin{pmatrix} x \\ y \end{pmatrix}$$

for all $x, y \in \mathbb{R}$. Is there any part of your conjecture that you do not know how to verify? Explain.