Problem Set 7: Due Monday, February 25

Problem 1: In each of the following cases, determine if the given function $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation. If yes, explain why. If no, provide an explicit counterexample.

a. $T\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) = \begin{pmatrix} 2x + 7y \\ 5x - 4y \end{pmatrix}$

b. $T\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) = \begin{pmatrix} xy \\ x + y \end{pmatrix}$

c. $T\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) = \begin{pmatrix} y\sin^2(x^3) + y\cos^2(x^3)) \\ y \end{pmatrix}$

Problem 2: Consider the linear transformation $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ given by

$$T\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) = \begin{pmatrix} x \\ -x + y \end{pmatrix}$$

Plot the values of at least 4 points and where T sends them, and then use that to describe the action of T geometrically.

Problem 3: Consider the linear transformation $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ given by

$$T\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) = \begin{pmatrix} 2x - y \\ -5x + 3y \end{pmatrix}.$$

Show that

$$\begin{pmatrix} -18 \\ 47 \end{pmatrix} \in \text{range}(T)$$

by explicitly finding $\vec{v} \in \mathbb{R}^2$ with

$$T(\vec{v}) = \begin{pmatrix} -18 \\ 47 \end{pmatrix}.$$

Problem 4: Show that the linear transformation $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ given by

$$T\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) = \begin{pmatrix} x + 2y \\ 3x + 6y \end{pmatrix}$$

is not injective and not surjective.

Problem 5: Suppose that $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ and $S\colon \mathbb{R}^2 \to \mathbb{R}^2$ are both linear transformations. Show that $T \circ S\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation.