Problem Set 5: Due Friday, February 15

Problem 1: Let $f\colon \mathbb{R} \to \mathbb{R}$ be given by $f(x) = x^3 - 8x$. Show that f is not injective.

Problem 2: Define a function $f\colon \{0,1,2,3,4\} \to \{0,1,2,3,4\}$ as follows. On input $n \in \{0,1,2,3,4\}$, let $f(n)$ be the remainder that arises when you divide the number $3n$ by 5. Is f injective, surjective, both, or neither? Explain.

Problem 3: Let $A = \{1,2\}$. Given an example, with explanation, of two functions $f\colon A \to A$ and $g\colon A \to A$ such that $f \circ g \neq g \circ f$.

Problem 4: Determine if the three lines $2x + y = 5$, $7x - 2y = 1$, and $-5x + 3y = 4$ intersect. Explain your reasoning using a few sentences.

Problem 5: Let

$$A = \left\{ \begin{pmatrix} 3 \\ -1 \end{pmatrix} + c \cdot \begin{pmatrix} 1 \\ 4 \end{pmatrix} : c \in \mathbb{R} \right\} \qquad \text{and} \qquad B = \left\{ \begin{pmatrix} 5 \\ 7 \end{pmatrix} + c \cdot \begin{pmatrix} 1 \\ 4 \end{pmatrix} : c \in \mathbb{R} \right\}.$$

In this problem, we will work through the outline of how to show that $A = B$ via a double containment proof.

a. Let's show that $A \subseteq B$. Let $\vec{u} \in A$ be arbitrary. By definition of A, we can fix $c \in \mathbb{R}$ with

$$\vec{u} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} + c \cdot \begin{pmatrix} 1 \\ 4 \end{pmatrix}.$$

To show that $\vec{u} \in B$, we have to fill in the blank of

$$\vec{u} = \begin{pmatrix} 5 \\ 7 \end{pmatrix} + \underline{\qquad} \cdot \begin{pmatrix} 1 \\ 4 \end{pmatrix}$$

with a real number. What real number works? Justify your choice.

b. Let's show that $B \subseteq A$. Let $\vec{u} \in B$ be arbitrary. By definition of B, we can fix $c \in \mathbb{R}$ with

$$\vec{u} = \begin{pmatrix} 5 \\ 7 \end{pmatrix} + c \cdot \begin{pmatrix} 1 \\ 4 \end{pmatrix}.$$

To show that $\vec{u} \in A$, we have to fill in the blank of

$$\vec{u} = \begin{pmatrix} 3 \\ -1 \end{pmatrix} + \underline{\qquad} \cdot \begin{pmatrix} 1 \\ 4 \end{pmatrix}$$

with a real number. What real number works? Justify your choice.

Problem 6: Verify part (8) of Proposition 2.2.1. That is, show that for all $\vec{v} \in \mathbb{R}^2$ and all $r, s \in \mathbb{R}$, we have $(r + s) \cdot \vec{v} = r \cdot \vec{v} + s \cdot \vec{v}$.