

Problem Set 16: Due Monday, April 15

Problem 1: Working in \mathcal{P}_3 , consider the following polynomial functions:

- $f_1(x) = x^3 + 2x^2 + x$.
- $f_2(x) = -3x^3 - 5x^2 + x + 2$.
- $f_3(x) = x^2 - x + 1$.
- $g(x) = x^3 + 8x^2 + 7$.

Is $g \in \text{Span}(f_1, f_2, f_3)$? Explain.

Problem 2: Use Gaussian Elimination to classify for which values of $h, k \in \mathbb{R}$ the system

$$\begin{aligned}x + hy &= 2 \\ 4x + 8y &= k\end{aligned}$$

has each of the following: (i) no solution, (ii) one solution, and (iii) infinitely many solutions.

Problem 3: Given $b_1, b_2, b_3 \in \mathbb{R}$, determine necessary and sufficient conditions so that

$$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \in \text{Span} \left(\begin{pmatrix} 0 \\ 1 \\ 5 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix} \right)$$

is true.

Problem 4: Does

$$\text{Span} \left(\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right) = \mathbb{R}^3?$$

Explain.

Problem 5: Let V be the vector space of all 2×2 matrices. Does

$$\text{Span} \left(\begin{pmatrix} 1 & 1 \\ 2 & 0 \end{pmatrix}, \begin{pmatrix} 2 & 3 \\ 7 & 2 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 2 & 6 \end{pmatrix} \right) = V?$$

Explain.

Problem 6: Determine exact conditions (that is, conditions that are both necessary and sufficient) on $a, b, c, d \in \mathbb{R}$ such that

$$\begin{aligned}x - 3y &= a \\ 3x + y &= b \\ x + 7y &= c \\ 2x + 4y &= d\end{aligned}$$

has a solution.