Problem Set 15: Due Friday, April 12

Problem 1: Let V be a vector space, and let W be a subspace of V. Recall that

$$V \backslash W = \{\vec{v} \in V : \vec{v} \notin W\},$$

i.e. $V \backslash W$ is the set of elements of V that are *not* in W. Is $V \backslash W$ always a subspace of V? Sometimes a subspace of V? Never a subspace of V? Explain.

Problem 2: Use Gaussian Elimination to solve the following system:

$$\begin{array}{rcrcrcr} x & & & - & z & = & 0 \\ 3x & + & y & & & = & 1 \\ -x & + & y & + & z & = & 4. \end{array}$$

Problem 3: Find the coefficients $a, b, c \in \mathbb{R}$ so that the graph of $f(x) = ax^2 + bx + c$ passes through the points $(1, 2)$, $(-1, 6)$, and $(2, 3)$.

Problem 4: Is

$$\begin{pmatrix} 20 \\ 0 \\ 5 \\ 10 \end{pmatrix} \in \text{Span}\left(\begin{pmatrix} 0 \\ 2 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 4 \\ -2 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \\ -1 \end{pmatrix} \right)?$$

Explain.

Problem 5: Give a parametric description of the solution set of the following system:

$$\begin{array}{rcrcrcrcr} x & + & 2y & - & z & & & = & 3 \\ 2x & + & y & & & + & w & = & 4 \\ x & - & y & + & z & + & w & = & 1. \end{array}$$

Problem 6: Let $\mathcal{D}$ be the vector space of all differentiable functions $f \colon \mathbb{R} \to \mathbb{R}$. Let $f_1 \colon \mathbb{R} \to \mathbb{R}$ be the function $f_1(x) = \sin^2 x$ and let $f_2 \colon \mathbb{R} \to \mathbb{R}$ be the function $f_2(x) = \cos^2 x$. Finally, let $W = \text{Span}(f_1, f_2)$, and notice that W is a subspace of $\mathcal{D}$. Determine, with explanation, whether the following functions are elements of W.

a. The function $g_1 \colon \mathbb{R} \to \mathbb{R}$ given by $g_1(x) = 3$.
b. The function $g_2 \colon \mathbb{R} \to \mathbb{R}$ given by $g_2(x) = x^2$.
c. The function $g_3 \colon \mathbb{R} \to \mathbb{R}$ given by $g_3(x) = \sin x$.
d. The function $g_4 \colon \mathbb{R} \to \mathbb{R}$ given by $g_4(x) = \cos 2x$.