Problem Set 11: Due Monday, March 11

Problem 1: Let

$$\alpha = \left(\begin{pmatrix} -1 \\ 2 \end{pmatrix}, \begin{pmatrix} 5 \\ 1 \end{pmatrix} \right).$$

a. Show that α is a basis for $\mathbb{R}^2$.
b. Compute $\left[\begin{pmatrix} 5 \\ 1 \end{pmatrix} \right]_\alpha$.
c. Compute $\left[\begin{pmatrix} 8 \\ 17 \end{pmatrix} \right]_\alpha$.
In each part, briefly explain how you carried out your computation.

Problem 2: Consider the unique linear transformation $T \colon \mathbb{R}^2 \to \mathbb{R}^2$ with

$$[T] = \begin{pmatrix} 6 & -7 \\ 4 & -5 \end{pmatrix}.$$

Let $\alpha = (\vec{u}_1, \vec{u}_2)$ where

$$\vec{u}_1 = \begin{pmatrix} 5 \\ 3 \end{pmatrix} \quad \text{and} \quad \vec{u}_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}.$$

In this problem, we compute $[T]_\alpha$ directly from the definition.
a. Show that $\alpha = (\vec{u}_1, \vec{u}_2)$ is a basis of $\mathbb{R}^2$.
b. Determine $T(\vec{u}_1)$ and then use this to compute $[T(\vec{u}_1)]_\alpha$.
c. Determine $T(\vec{u}_2)$ and then use this to compute $[T(\vec{u}_2)]_\alpha$.
d. Using parts (b) and (c), determine $[T]_\alpha$.

Problem 3: With the same setup as Problem 2, compute $[T]_\alpha$ using Proposition 3.2.6.

Problem 4: Again, use the same setup as in Problem 2. Let

$$\vec{v} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}.$$

In this problem, we compute $[T(\vec{v})]_\alpha$ in two different ways.
a. First determine $T(\vec{v})$, and then use this to compute $[T(\vec{v})]_\alpha$.
b. First determine $[\vec{v}]_\alpha$, and then multiply the result by your matrix $[T]_\alpha$ to compute $[T(\vec{v})]_\alpha$.

Problem 5: Consider the unique linear transformation $T \colon \mathbb{R}^2 \to \mathbb{R}^2$ with

$$[T] = \begin{pmatrix} 3 & 2 \\ 4 & -1 \end{pmatrix}.$$

Let $\alpha = (\vec{u}_1, \vec{u}_2)$ where

$$\vec{u}_1 = \begin{pmatrix} -4 \\ -2 \end{pmatrix} \quad \text{and} \quad \vec{u}_2 = \begin{pmatrix} 9 \\ 4 \end{pmatrix}.$$

Compute $[T]_\alpha$ using any method.

Problem 6: Let A and B be 2×2 matrices. Assume that $A\vec{v} = B\vec{v}$ for all $\vec{v} \in \mathbb{R}^2$. Show that $A = B$.