Written Assignment 9 : Due Wednesday, April 27

Problem 1: Find values of c and d such that the matrix

$$\begin{bmatrix} 3 & 1 \\ c & d \end{bmatrix}$$

has both 4 and 7 as eigenvalues. You should show the derivation for how you arrived at your choice.

Problem 2: Let A be an $n \times n$ idempotent matrix.
a. Show that $Nul(A) \cap Col(A) = \{\mathbf{0}\}$.
b. Show that for every $\mathbf{v} \in \mathbb{R}^n$, there exists $\mathbf{u} \in Nul(A)$ and $\mathbf{w} \in Col(A)$ with $\mathbf{v} = \mathbf{u} + \mathbf{w}$.

Problem 3: Define a sequence of numbers as follows. Let $g_0 = 0$, $g_1 = 1$, and $g_n = \frac{1}{2}(g_{n-1} + g_{n-2})$ for $n \geq 2$. In other words, the n^{th} term of the sequence is the average of the two previous terms.
a. Find a general equation for g_n.
b. As n gets large, the values of g_n approach a fixed number. Find that number.
c. Suppose that you change the initial starting values of g_0 and g_1. As n gets large, must the terms of the sequence astill approach a fixed number? If so, explain why and determine that number in terms of g_0 and g_1. If not, find an example where the terms of sequence do not approach one fixed number.