Written Assignment 8 : Due Wednesday, April 20

Problem 1:
a. Give an example of a *nonzero* 2×2 matrix whose only eigenvalue is 0.
b. Let A be a 2×2 matrix and suppose that 0 is the only eigenvalue of A. Show that A^2 is the zero matrix.

Problem 2: Suppose that A is an $n \times n$ idempotent matrix (recall this means that $A^2 = A$).
a. Show that the only possible eigenvalues of A are 0 and 1.
b. Suppose also that $A \neq I$. Show that 0 is in fact an eigenvalue of A.

Problem 3: Let A be an $n \times n$ matrix and suppose that every vector in $\mathbb{R}^n$ is an eigenvector of A (but do not assume that they all correspond to the same eigenvalue). Show that there exists a scalar c such that $A = cI$.