Written Assignment 6 : Due Wednesday, March 16

Problem 1: Define $T\colon \mathbb{P}_2 \to \mathbb{R}$ by $T(p(t)) = \int_0^1 p(t)\ dt$. You know from Calculus that T is a linear transformation. Find, with proof, a basis for $\ker(T)$ and a basis for $\text{range}(T)$.

Problem 2: Let V be a vector space and suppose that $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k\}$ is linearly independent. Suppose that $\{\mathbf{v}_1 + \mathbf{w}, \mathbf{v}_2 + \mathbf{w}, \ldots, \mathbf{v}_k + \mathbf{w}\}$ is linearly dependent. Show that $\mathbf{w} \in Span\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k\}$.

Problem 3: Suppose that V and W are vector spaces and that $T\colon V \to W$ is a linear transformation.
a. Show that if $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k\}$ is a linearly dependent set in V, then $\{T(\mathbf{v}_1), T(\mathbf{v}_2), \ldots, T(\mathbf{v}_k)\}$ is a linearly dependent set in W.
b. Suppose that T is one-to-one. Show that if $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k\}$ is a linearly independent set in V, then $\{T(\mathbf{v}_1), T(\mathbf{v}_2), \ldots, T(\mathbf{v}_k)\}$ is a linearly independent set in W.
c. Suppose that T is both one-to-one and onto. Show that if $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k\}$ is a basis for V, then $\{T(\mathbf{v}_1), T(\mathbf{v}_2), \ldots, T(\mathbf{v}_k)\}$ is a basis for W.