Written Assignment 10 : Due Wednesday, May 11

Note: Let U be $n \times n$ matrix. Recall that U is an *orthogonal* matrix if it is has orthonormal columns. We know that this is equivalent to saying that $U^T U = I$.

Problem 1: Let U be an orthogonal matrix.
a. Show that U^T is an orthogonal matrix.
b. Show that U has orthonormal rows.

Problem 2: Show that the only possible (real) eigenvalues of an orthogonal matrix are 1 and -1.

Problem 3: Let W be a subspace of $\mathbb{R}^n$ with $\dim W = k$. Show that $\dim W^\perp = n - k$.
Hint: We know that every subspace of $\mathbb{R}^n$ has an orthogonal basis (because we know every subspace has a basis, and we can use Gram-Schmidt to get an orthogonal one). Start by taking an orthogonal basis for each of W and $W^\perp$.