Written Assignment 1 : Due Wednesday, February 2

Problem 1: If A is the augmented matrix of a linear system, and you obtain B from A by multiplying a column of A by a nonzero constant, must the corresponding systems have the same solution set? You should either argue that this operation always works, or produce a specific counterexample.

Problem 2: Explain why, no matter what values a, b, c are used, the following two matrices are not row equivalent.

$$\begin{bmatrix} 1 & 1 & 2 \\ -2 & 0 & -1 \\ 1 & 3 & 5 \end{bmatrix} \qquad \begin{bmatrix} 4 & 2 & 1 \\ a & -1 & 0 \\ b & c & 3 \end{bmatrix}$$

Hint: It is impossible to try all conceivable sequences of elementary row operations. What do you know about two matrices that are row equivalent?

Problem 3: Suppose that you are given a linear system

$$\begin{aligned} a_{1,1}x_1 + a_{1,2}x_2 + \cdots + a_{1,n}x_n &= 0 \\ a_{2,1}x_1 + a_{2,2}x_2 + \cdots + a_{2,n}x_n &= 0 \\ &\vdots \\ a_{m,1}x_1 + a_{m,2}x_2 + \cdots + a_{m,n}x_n &= 0 \end{aligned}$$

Suppose that $(s_1, s_2, \ldots, s_n)$ and $(t_1, t_2, \ldots, t_n)$ are both solutions to this linear system. Explain why $(s_1 + t_1, s_2 + t_2, \ldots, s_n + t_n)$ is also a solution.