Writing Assignment 8: Due Wednesday, November 9

Problem 1: Let V be a vector space. Suppose that U and W are both subspaces of V. You showed in Writing Assignment 7 that $U \cup W$ might not be a subspace of V. Instead, let

$$U + W = \{\vec{v} \in V : \text{There exists } \vec{u} \in U \text{ and } \vec{w} \in W \text{ with } \vec{v} = \vec{u} + \vec{w}\}.$$

That is, $U + W$ is the set of all vectors in V that can be written as the sum of an element of U and an element of W. Show that $U + W$ is a subspace of V.

Problem 2: Corollary 4.2.5 says that elementary row operations preserve the solution set of the corresponding linear systems. What about column operations? Consider multiplying a column of a linear system by a nonzero number. Do we necessarily preserve the solution set of the system by doing this? As always, you must explain if your answer is yes, or you must provide a specific counterexample (with justification) if your answer is no.

Problem 3: Show that for all $a, b, c \in \mathbb{R}$, the matrices

$$\begin{pmatrix} 4 & 2 & 1 \\ a & -1 & 0 \\ b & c & 3 \end{pmatrix}$$

and

$$\begin{pmatrix} 1 & 1 & 2 \\ -2 & 0 & -1 \\ 1 & 3 & 5 \end{pmatrix}$$

are not row equivalent, i.e. there does not exist a sequence of elementary row operations that turns the first matrix into the second matrix.