Writing Assignment 7: Due Friday, November 4

Problem 1: Let V be a vector space. Suppose that U and W are both subspaces of V.
a. Let $U \cap W$ be the intersection of U and W, i.e. $U \cap W = \{\vec{v} \in V : \vec{v} \in U \text{ and } \vec{v} \in W\}$. Show that $U \cap W$ is a subspace of V.
b. Let $U \cup W$ be the union of U and W, i.e. $U \cup W = \{\vec{v} \in V : \vec{v} \in U \text{ or } \vec{v} \in W\}$. By constructing an explicit example (with justification), show that $U \cup W$ might *not* be a subspace of V.
c. Give an example (with justification) of a vector space V together with two subspaces U and W of V such that $U \cup W$ is a subspace of V.

Problem 2: We know that $\{\vec{0}\}$ and $\mathbb{R}^2$ are both subspaces of $\mathbb{R}^2$. We also know that $\mathsf{Span}(\vec{u})$ is a subspace of $\mathbb{R}^2$ for each nonzero $\vec{u} \in \mathbb{R}^2$. Show that these are the only subspaces of $\mathbb{R}^2$.
Hint: Take an arbitrary subspace $W \subseteq \mathbb{R}^2$. By definition of a subspace, we know that $\vec{0} \in W$. If there are no other vectors in W, then $W = \{\vec{0}\}$. Otherwise, W contains a nonzero vector. Fix such a vector, and think about what you can say from here.