Writing Assignment 4: Due Wednesday, September 28

Problem 1: Let $\vec{u}, \vec{w} \in \mathbb{R}^2$. Suppose that $\vec{w} \in \mathsf{Span}(\vec{u})$ and $\vec{w} \neq \vec{0}$. Show that $\mathsf{Span}(\vec{w}) = \mathsf{Span}(\vec{u})$.

Problem 2: Let $\vec{u}_1, \vec{u}_2, \vec{u}_3 \in \mathbb{R}^2$. Show that at least one of the $\vec{u}_i$ is in the span of the other two. That is show that either $\vec{u}_1 \in \mathsf{Span}(\vec{u}_2, \vec{u}_3)$, or $\vec{u}_2 \in \mathsf{Span}(\vec{u}_1, \vec{u}_3)$, or $\vec{u}_3 \in \mathsf{Span}(\vec{u}_1, \vec{u}_2)$.

Problem 3: Let $T \colon \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation. Recall that

$$\mathsf{range}(T) = \{\vec{w} \in \mathbb{R}^2 : \text{There exists } \vec{v} \in \mathbb{R}^2 \text{ with } \vec{w} = T(\vec{v})\}.$$

Notice that $\vec{0} \in \mathsf{range}(T)$ because we know that $T(\vec{0}) = \vec{0}$ by Proposition 2.4.2.
a. Show that if $\vec{w}_1, \vec{w}_2 \in \mathsf{range}(T)$, then $\vec{w}_1 + \vec{w}_2 \in \mathsf{range}(T)$.
b. Show that if $\vec{w} \in \mathsf{range}(T)$ and $c \in \mathbb{R}$, then $c\vec{w} \in \mathsf{range}(T)$.