Writing Assignment 10: Due Wednesday, November 30

Problem 1: Let V be a vector space, and let U and W be subspaces of V. Recall from Writing Assignment 8 that

$$U + W = \{\vec{v} \in V : \text{There exists } \vec{u} \in U \text{ and } \vec{w} \in W \text{ with } \vec{v} = \vec{u} + \vec{w}\}.$$

Assume that $(\vec{u}_1, \dots, \vec{u}_m)$ is a basis of U and that $(\vec{w}_1, \dots, \vec{w}_n)$ is a basis of W.
a. Show that $U + W = \mathsf{Span}(\vec{u}_1, \dots, \vec{u}_m, \vec{w}_1, \dots, \vec{w}_n)$.
b. Show that $\dim(U + W) \leq \dim(U) + \dim(W)$.

Problem 2: Let V and W be vector spaces. Suppose that $T \colon V \to W$ is an injective linear transformation and that $(\vec{u}_1, \vec{u}_2, \dots, \vec{u}_n)$ is a linearly independent sequence in V. Show that $(T(\vec{u}_1), T(\vec{u}_2), \dots, T(\vec{u}_n))$ is a linearly independent sequence in W.