

Writing Assignment 1: Due Wednesday, August 31

Problem 1: Determine whether the following statements are true or false. In each case, explain your reasoning thoroughly in complete sentences.

- a. There exists $m, n \in \mathbb{Z}$ such that $34m + 30n = 2$.
- b. There exists $m, n \in \mathbb{N}$ such that $5m + 9n = 16$.

Problem 2: Define a function $f: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ by letting

$$f\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) = \begin{pmatrix} x - y \\ x + y \end{pmatrix}.$$

Think of f as transforming the plane as we discussed in class, and as illustrated on p. 9 of the book. As discussed there, it appears that f rotates the plane 45° counterclockwise while simultaneously scaling the plane by a factor of $\sqrt{2}$. In this problem, you will verify some of these statements.

- a. Show that for all $\vec{v} \in \mathbb{R}^2$, we have $\|f(\vec{v})\| = \sqrt{2} \cdot \|\vec{v}\|$, where $\|\vec{v}\|$ is the length of \vec{v} .
- b. Show that for all nonzero $\vec{v} \in \mathbb{R}^2$, the angle between \vec{v} and $f(\vec{v})$ is 45° .
- c. Parts (a) and (b) give two possibilities of $f(\vec{v})$, since it might be clockwise or counterclockwise relative to \vec{v} . Think about how you check that it is counterclockwise, and try to verify it.

Hint: Think about using the dot product and cross product for parts (b) and (c).