Problem Set 7: Due Monday, September 26

Problem 1: Consider the unique linear transformation $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ with

$$T\left(\begin{pmatrix}9\\4\end{pmatrix}\right) = \begin{pmatrix}1\\-5\end{pmatrix} \qquad \text{and} \qquad T\left(\begin{pmatrix}2\\1\end{pmatrix}\right) = \begin{pmatrix}-2\\3\end{pmatrix}.$$

Determine, with explanation, the value of

$$T\left(\begin{pmatrix}6\\2\end{pmatrix}\right).$$

Problem 2: Show that the linear transformation $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ given by

$$T\left(\begin{pmatrix}x\\y\end{pmatrix}\right) = \begin{pmatrix}x+2y\\3x+6y\end{pmatrix}$$

is not injective and not surjective.

Problem 3: Consider the linear transformation $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ given by

$$T\left(\begin{pmatrix}x\\y\end{pmatrix}\right) = \begin{pmatrix}x\\-x+y\end{pmatrix}.$$

Plot the values of at least 4 points and where T sends them, and then use that to describe the action of T geometrically.

Problem 4: Suppose that $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ and $S\colon \mathbb{R}^2 \to \mathbb{R}^2$ are both linear transformations. Show that $T \circ S\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation.

Problem 5: Compute

$$\begin{pmatrix}4 & 3\\-7 & 1\end{pmatrix}\begin{pmatrix}1\\5\end{pmatrix}.$$

Describe what your computation means in terms of a linear transformation. Use Problem 1 above as a guide.

Problem 6: Consider the unique linear transformation $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ with

$$T\left(\begin{pmatrix}1\\-1\end{pmatrix}\right) = \begin{pmatrix}1\\4\end{pmatrix} \qquad \text{and} \qquad T\left(\begin{pmatrix}-2\\3\end{pmatrix}\right) = \begin{pmatrix}2\\7\end{pmatrix}.$$

What is $[T]$? Explain.