Problem Set 5: Due Friday, September 16

Problem 1: Define $f\colon \mathbb{R} \to \mathbb{R}$ by $f(x) = x^3 + 3x + 1$. It can be shown that f is bijective (you do *not* need to do this). What is $f^{-1}(15)$? Justify your answer.

Problem 2: Let $A = \{0, 1, 2, \dots, 10\}$. Define a function $f\colon A \to A$ by letting $f(n)$ be the remainder that arises when you divide the number $7n + 3$ by 11. It can be shown that f is bijective (you do *not* need to do this). What is $f^{-1}(4)$? Justify your answer.

Problem 3: Determine if the three lines described by the equations $2x+y = 5$, $7x-2y = 1$, and $-5x+3y = 4$ intersect. That is, determine whether there exists a point that lies on all three of the corresponding lines. Explain your reasoning using a few sentences.

Problem 4: Verify part (8) of Proposition 2.2.1. That is, show that for all $\vec{v} \in \mathbb{R}^2$ and all $r, s \in \mathbb{R}$, we have $(r + s) \cdot \vec{v} = r \cdot \vec{v} + s \cdot \vec{v}$.

Problem 5: Fill in the blanks below with appropriate phrases so that the result is a correct proof of the following statement:

$$\text{Let } \vec{u}, \vec{w} \in \mathbb{R}^2 \text{ and suppose that } \vec{w} \in \mathsf{Span}(\vec{u}). \text{ We then have } \mathsf{Span}(\vec{w}) \subseteq \mathsf{Span}(\vec{u}).$$

As in previous assignments, write up the whole proof, not just the pieces that go into the blanks. Also, fill in as much as necessary in the blanks so that no steps are omitted. Finally, if you would prefer to write the argument differently in your words, please feel free to do so.

Let $\vec{v} \in \mathsf{Span}(\vec{w})$ be arbitrary. By assumption, we know that $\vec{w} \in \mathsf{Span}(\vec{u})$, so we can ________________. Since $\vec{v} \in \mathsf{Span}(\vec{w})$, we can ________________. Now notice that $\vec{v} =$________________. Since ________________$\in \mathbb{R}$, we conclude that $\vec{v} \in \mathsf{Span}(\vec{u})$.

Problem 6: Let $\vec{u}, \vec{v}_1, \vec{v}_2 \in \mathbb{R}^2$, and suppose that $\vec{v}_1, \vec{v}_2 \in \mathsf{Span}(\vec{u})$. Show that if $\vec{v}_1 \neq \vec{0}$, then there exists $r \in \mathbb{R}$ with $\vec{v}_2 = r \cdot \vec{v}_1$.
Note: You have to use the fact that $\vec{v}_1 \neq \vec{0}$ somewhere, so explicitly point out where you are using that assumption.