Problem Set 3: Due Friday, September 9

Problem 1: Consider the following statement:

If $a \in \mathbb{Z}$ and $3a + 5$ is even, then a is odd.

a. Write down the contrapositive of the given statement.
b. Show that the original statement is true by proving that the contrapositive is true.

Problem 2: Let $A = \{\sin x : x \in \mathbb{R}\}$.
a. In class, we talked about how we could always turn a parametric description of a set into our other description (by carving out of a bigger set) by using a "there exists" quantifier. Do that for our set A above.
b. Find another way to describe A by carving it out of a set using a property without any quantifiers. Briefly explain why your set is equal (no need to give a formal proof).

Problem 3: Describe the set $\{x \in \mathbb{R} : x^2 - 3x - 4 > 0\}$ in another way by writing it as a union of two sets with simpler descriptions. Briefly explain why your set is equal.

Problem 4: Let $A = \{6n : n \in \mathbb{N}\} \cap \{10n : n \in \mathbb{N}\}$.
a. Write down the smallest 3 elements of A, and briefly explain how you determined them.
b. Make a conjecture about how to describe A parametrically (no need to prove this conjecture).

Problem 5: Let $A = \{12n - 7 : n \in \mathbb{Z}\}$ and let $B = \{4n + 1 : n \in \mathbb{Z}\}$.
a. Show that $B \not\subseteq A$.
b. Fill in the blanks below with appropriate phrases so that the result is a correct proof of the statement that $A \subseteq B$.

Let $a \in A$ be arbitrary. By definition of A, we can ________________. Now notice that $a =$ ________________.
Since ________________ $\in \mathbb{Z}$, we conclude that $a \in B$.

Problem 6: Let $A = \{x^2 + 5 : x \in \mathbb{R}\}$ and let $B = \{y \in \mathbb{R} : y \geq 5\}$. In this problem, we show that $A = B$ by doing a double containment proof.
a. Prove that $A \subseteq B$.
b. Notice that $5, 9, 10, 42 \in B$. Show that each of $5, 9, 10, 42 \in A$ by explicitly finding a value of $x \in \mathbb{R}$ with $x^2 + 5 = 5$, then finding a value of $x \in \mathbb{R}$ with $x^2 + 5 = 9$, etc.
c. Let $y \in B$ in arbitrary. Following the pattern that you see in part (b), what value of $x \in \mathbb{R}$ do you think will demonstrate that $y \in A$?
d. Using your idea from part (c), write a proof showing that $B \subseteq A$. At some point you will need to use the fact that elements of B are greater than or equal to 5, so be sure to point out where that is important!