Problem Set 21: Due Monday, December 5

Problem 1: Consider the matrix

$$A = \begin{pmatrix} 3 & 2 \\ 5 & 3 \end{pmatrix}.$$

We know from Proposition 2.7.9 that A is invertible, and we also know a formula for the inverse. Now compute A^{-1} using our new method by applying elementary row operations to the matrix

$$\begin{pmatrix} 3 & 2 & 1 & 0 \\ 5 & 3 & 0 & 1 \end{pmatrix}.$$

Problem 2: Determine whether each of the following matrices is invertible, and if so, find the inverse.

a. $\begin{pmatrix} 1 & 1 & 3 \\ 0 & 2 & 4 \\ -1 & 1 & 0 \end{pmatrix}$

b. $\begin{pmatrix} 0 & 4 & 4 \\ 1 & -2 & 0 \\ 3 & -4 & 2 \end{pmatrix}$

c. $\begin{pmatrix} 0 & 1 & 5 \\ 0 & -2 & 4 \\ 2 & 3 & -2 \end{pmatrix}$

Problem 3: Either prove or find a counterexample: If A and B are invertible $n \times n$ matrices, then $A + B$ is invertible.

Problem 4: Consider the 2×3 matrix

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}.$$

a. Show that A has no left inverse, i.e. that there does not exist a 3×2 matrix B with $BA = I_3$ (the 3×3 identity matrix).

b. Show that A has infinitely many right inverses, i.e. that there exist infinitely many 3×2 matrices C with $AC = I_2$ (the 2×2 identity matrix).

Problem 5: Suppose that

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 5.$$

Find, with explanation, the value of

$$\begin{vmatrix} -d & -e & -f \\ 2g + 3a & 2h + 3b & 2i + 3c \\ a & b & c \end{vmatrix}.$$