Problem Set 18: Due Friday, November 18

Problem 1: Let
$$\alpha = \left(\begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}, \begin{pmatrix} -1 \\ -3 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 8 \\ 9 \end{pmatrix} \right).$$
a. Show that α is a basis of $\mathbb{R}^3$.
b. Determine
$$\left[\begin{pmatrix} 1 \\ 5 \\ -5 \end{pmatrix} \right]_\alpha.$$

Problem 2: Consider the following elements of $\mathcal{P}_3$:

- $f_1(x) = x^3$.
- $f_2(x) = x^3 + x^2$.
- $f_3(x) = x^3 + x^2 + x$.
- $f_4(x) = x^3 + x^2 + x + 1$.

Let $\alpha = (f_1, f_2, f_3, f_4)$.
a. Show that α is a basis of $\mathcal{P}_3$.
b. Let $g(x) = 3x^3 + 7x^2 + 7x - 2$. Determine $[g]_\alpha$.

Problem 3: Working in $\mathbb{R}^4$, let
$$W = \mathsf{Span}\left(\begin{pmatrix} 0 \\ 0 \\ 1 \\ 3 \end{pmatrix}, \begin{pmatrix} 4 \\ 5 \\ 2 \\ 7 \end{pmatrix}, \begin{pmatrix} 7 \\ 8 \\ 0 \\ 1 \end{pmatrix} \right).$$
Explain why $\dim(W) = 3$.

Problem 4: Working in $\mathbb{R}^4$, let
$$W = \mathsf{Span}\left(\begin{pmatrix} 1 \\ 3 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 6 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 3 \\ 9 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \\ -1 \\ 7 \end{pmatrix}, \begin{pmatrix} -4 \\ -7 \\ 0 \\ -3 \end{pmatrix} \right).$$
a. Find (with explanation) a basis for W.
b. Determine $\dim(W)$.

Problem 5: Let V be the vector space of all 2×2 matrices. Let
$$W = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in V : b = c \right\}.$$
It can be checked that W is a subspace of V (no need to do this). Find a basis for W, and determine $\dim(W)$.