Problem Set 17: Due Monday, November 14

Problem 1: Determine whether
$$\left(\begin{pmatrix} 1 \\ -3 \\ 5 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ 4 \end{pmatrix}, \begin{pmatrix} 4 \\ -4 \\ 14 \end{pmatrix}\right)$$
is a linearly independent sequence in $\mathbb{R}^3$.

Problem 2: By setting up a system and using Gaussian Elimination, find one specific example of nontrivial linear combination of
$$\left(\begin{pmatrix} 0 \\ 1 \\ 3 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 2 \\ -1 \end{pmatrix}, \begin{pmatrix} -8 \\ 2 \\ -2 \\ 2 \end{pmatrix}, \begin{pmatrix} 6 \\ -1 \\ 9 \\ 5 \end{pmatrix}\right)$$
giving $\vec{0}$.

Problem 3: Consider the following three functions in the vector space $\mathcal{P}_2$:

- $f_1(x) = 9x^2 - x + 3$.
- $f_2(x) = 3x^2 - 2x + 5$.
- $f_3(x) = -5x^2 + x + 1$.

Is (f_1, f_2, f_3) linearly independent? Explain.

Problem 4: Consider the following three functions in the vector space $\mathcal{F}$:

- $f_1(x) = 2^x$.
- $f_2(x) = x^2$.
- $f_3(x) = x - 2$.

Is (f_1, f_2, f_3) linearly independent? Explain.

Problem 5: Find a sequence $(\vec{u}_1, \vec{u}_2, \vec{u}_3, \vec{u}_4)$ of vectors in $\mathbb{R}^3$ such that whenever we omit a vector, the resulting 3 are linearly independent. You should justify why your sequence has this property.

Problem 6: In Problem 4 on Problem Set 14, you showed that
$$W = \left\{ \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \in \mathbb{R}^3 : a_1 + a_2 + a_3 = 0 \right\}$$
was a subspace of $\mathbb{R}^3$. Show that
$$W = \mathsf{Span}\left(\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}\right)$$
by giving a double containment proof.

Aside: Using this result, we can instead apply Proposition 4.1.16 to conclude that W is a subspace of $\mathbb{R}^3$.