

## Problem Set 16: Due Friday, November 11

**Problem 1:** Working in  $\mathcal{P}_3$ , consider the following functions:

- $f_1(x) = x^3 + 2x^2 + x$ .
- $f_2(x) = -3x^3 - 5x^2 + x + 2$ .
- $f_3(x) = x^2 - x + 1$ .
- $g(x) = x^3 + 8x^2 + 7$ .

Is  $g \in \text{Span}(f_1, f_2, f_3)$ ? Explain.

**Problem 2:** Use Gaussian Elimination to classify for which values of  $h, k \in \mathbb{R}$  the system

$$\begin{array}{rcrcrcrcrcl} x & + & hy & = & 2 \\ 4x & + & 8y & = & k \end{array}$$

has each of the following: (i) no solution, (ii) one solution, and (iii) infinitely many solutions.

**Problem 3:** Given  $b_1, b_2, b_3 \in \mathbb{R}$ , determine necessary and sufficient conditions so that

$$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \in \text{Span} \left( \begin{pmatrix} 0 \\ 1 \\ 5 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix} \right)$$

is true.

**Problem 4:** Does

$$\text{Span} \left( \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right) = \mathbb{R}^3?$$

Explain.

**Problem 5:** Let  $V$  be the vector space of all  $2 \times 2$  matrices. Does

$$\text{Span} \left( \begin{pmatrix} 1 & 1 \\ 2 & 0 \end{pmatrix}, \begin{pmatrix} 2 & 3 \\ 7 & 2 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 2 & 6 \end{pmatrix} \right) = V?$$

Explain.

**Problem 6:** Let  $\mathcal{D}$  be the vector space of all differentiable functions  $f: \mathbb{R} \rightarrow \mathbb{R}$ . Let  $f_1: \mathbb{R} \rightarrow \mathbb{R}$  be the function  $f_1(x) = \sin^2 x$  and let  $f_2: \mathbb{R} \rightarrow \mathbb{R}$  be the function  $f_2(x) = \cos^2 x$ . Finally, let  $W = \text{Span}(f_1, f_2)$ , and notice that  $W$  is a subspace of  $\mathcal{D}$ . Determine, with explanation, whether the following functions are elements of  $W$ .

- The function  $g_1: \mathbb{R} \rightarrow \mathbb{R}$  given by  $g_1(x) = 3$ .
- The function  $g_2: \mathbb{R} \rightarrow \mathbb{R}$  given by  $g_2(x) = x^2$ .
- The function  $g_3: \mathbb{R} \rightarrow \mathbb{R}$  given by  $g_3(x) = \sin x$ .
- The function  $g_4: \mathbb{R} \rightarrow \mathbb{R}$  given by  $g_4(x) = \cos 2x$ .