

## Problem Set 14: Due Monday, October 31

**Problem 1:** Let  $V = \mathbb{R}^3$ , but with the following operations:

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

and

$$c \cdot \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} ca_1 \\ ca_2 \\ ca_3 \end{pmatrix}.$$

Show that there is no element of  $V$  that serves as  $\vec{0}$ . That is, show that there does not exist  $\vec{z} \in V$  such that  $\vec{v} + \vec{z} = \vec{v}$  for all  $\vec{v} \in V$ .

**Problem 2:** Let  $V = \mathbb{R}^2$ , but with the following operations:

$$\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} a_1 + b_1 \\ a_2 + b_2 \end{pmatrix}$$

and

$$c \cdot \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} ca_1 \\ a_2 \end{pmatrix}.$$

Also, let

$$\vec{0} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

Show that  $V$  is not a vector space by explicitly finding a counterexample to one of the 10 properties.

**Problem 3:** Let  $V$  be a vector space. Show that  $\vec{u} + (\vec{v} + \vec{w}) = \vec{w} + (\vec{v} + \vec{u})$  for all  $\vec{u}, \vec{v}, \vec{w} \in V$ . Carefully state what property you are using in every step of your argument.

**Problem 4:** Show that

$$\left\{ \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \in \mathbb{R}^3 : a_1 + a_2 + a_3 = 0 \right\}$$

is a subspace of  $\mathbb{R}^3$ .

**Problem 5:** Recall that  $\mathcal{P}$  is the vector space of all polynomial functions  $f: \mathbb{R} \rightarrow \mathbb{R}$ . Let  $W$  be the subset of  $\mathcal{P}$  consisting of those polynomials that have a nonnegative constant term (i.e. the constant term is greater than or equal to 0). Is  $W$  a subspace of  $\mathcal{P}$ ? Either prove or give a counterexample.

**Problem 6:** Let  $V$  be the vector space of all  $2 \times 2$  matrices. Show that

$$\begin{pmatrix} -2 & 7 \\ -1 & -9 \end{pmatrix} \in \text{Span} \left( \begin{pmatrix} 1 & 1 \\ 2 & -3 \end{pmatrix}, \begin{pmatrix} 3 & 0 \\ 5 & -4 \end{pmatrix} \right).$$