

## Problem Set 12: Due Friday, October 14

**Problem 1:** Find the eigenvalues of the matrix

$$\begin{pmatrix} 5 & -1 \\ -7 & 3 \end{pmatrix}.$$

**Problem 2:** Find the eigenvalues of the matrix

$$\begin{pmatrix} 1 & 8 \\ 2 & 1 \end{pmatrix},$$

and then find (at least) one eigenvector for each eigenvalue.

**Problem 3:** Find the eigenvalues of the matrix

$$\begin{pmatrix} 2 & -1 \\ 1 & 4 \end{pmatrix},$$

and then find (at least) one eigenvector for each eigenvalue.

**Problem 4** Find values for  $c$  and  $d$  such that the matrix

$$\begin{pmatrix} 3 & 1 \\ c & d \end{pmatrix}$$

has both 4 and 7 as eigenvalues. You should show the derivation for how you arrived at your choice.

**Problem 5:** Consider the unique linear transformation  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  with

$$[T] = \begin{pmatrix} 1 & 0 \\ 6 & -1 \end{pmatrix}.$$

Determine whether  $T$  is diagonalizable. If so, find an example of a basis  $\alpha = (\vec{u}_1, \vec{u}_2)$  of  $\mathbb{R}^2$  such that  $[T]_\alpha$  is a diagonal matrix, and determine  $[T]_\alpha$  in this case.

**Problem 6:** Consider the unique linear transformation  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  with

$$[T] = \begin{pmatrix} 3 & -1 \\ 1 & 5 \end{pmatrix}.$$

Determine whether  $T$  is diagonalizable. If so, find an example of a basis  $\alpha = (\vec{u}_1, \vec{u}_2)$  of  $\mathbb{R}^2$  such that  $[T]_\alpha$  is a diagonal matrix, and determine  $[T]_\alpha$  in this case.