Problem Set 1: Due Friday, September 2

Problem 1: Let $A = \{1, 2, 3, 4\}$. Suppose that $P(x, y)$ and $Q(x, y)$ are expressed by the following tables:

P	1	2	3	4
1	F	T	T	F
2	F	T	F	T
3	T	F	F	F
4	F	F	F	F

Q	1	2	3	4
1	F	T	T	T
2	F	T	T	T
3	T	F	T	F
4	F	F	T	T

Interpret the diagrams as follows. Given $x, y \in \{1, 2, 3, 4\}$, to determine the truth value of $P(x, y)$, go to row x and column y. For example, $P(1, 2)$ is true and $P(3, 3)$ is false. Determine, with careful explanation, whether each of the following are true or false.
a. There exist $x, y \in A$ with $\text{Not}(x = y)$, $P(x, y)$, and $P(y, x)$.
b. For all $x, y \in A$, we have $\text{Not}(Q(x, y))$.
c. For all $x, y \in A$, if $P(x, y)$, then $Q(x, y)$.
d. For all $x, y \in A$, either $Q(x, y)$ or $Q(y, x)$.
e. For all $x \in A$, there exists $y \in A$ with $\text{Not}(P(x, y))$.
f. There exists $x \in A$ such that for all $y \in A$, we have $P(x, y)$.
g. There exists $y \in A$ such that for all $x \in A$, we have $Q(x, y)$.

Problem 2: Given $x, y \in \mathbb{N}^+$, let $D(x, y)$ be true if x divides evenly into y. Express each of the following statements carefully using only $\mathbb{N}^+$ and D, our quantifiers "for all" and "there exists", and the connectives "=" "and", "or", "not", and "if...then...". In other words, express each of the following like the statements in Problem 1. You do *not* need to explain if the statements are true or false.
a. The number 1 divides evenly into all numbers.
b. Whenever x divides evenly into y, and y divides evenly into z, it follows that x divides evenly into z.
c. There is no number with the property that every number divides evenly into it.
d. The numbers 1 and 91 both divide evenly into 91, and no other number divides evenly into 91.

Problem 3: Let A be the set of all people. Given $x, y \in A$, let $M(x, y)$ be true if x has ever sent a text message to y, and be false otherwise. Express each of the following statements carefully using only A and M, our quantifiers "for all" and "there exists", and the connectives "=" "and", "or", "not", and "if...then...". In other words, express each of the following like the statements in Problem 1.
a. Everybody has sent a text message to somebody.
b. Somebody has never received a text message from anyone.
c. Somebody has sent a text message to (at least) two different people.
d. Somebody has sent a text message to each person that they have received a message from.

Problem 4:
a. Explain why "Not(P and Q)" is *not* equivalent to "Not(P) and Not(Q)", i.e. explain how the two statements can fail to have the same truth value.
b. Explain why "Not(P and Q)" is equivalent to "Not(P) or Not(Q)", i.e. explain why the two statements always have the same truth value.
Note: Feel free to use truth tables (with explanation) to help your argument, but you can also just argue using complete sentences.

Problem 5: Write the negation of each of the the following statements so that no "not" appears. You do *not* need to explain if the statements are true or false.

a. For all $x \in \mathbb{R}$, we have $e^x \neq 0$.
b. There exists $x \in \mathbb{R}$ such that for all $y \in \mathbb{R}$, we have $x + y^2 \geq 3$.
c. For all $x, y \in \mathbb{R}$, there exists $q \in \mathbb{Q}$ such that both $x < q$ and $q < y$.
d. There exists $y \in \mathbb{R}$ such that for all $x \in \mathbb{R}$, there exists $n \in \mathbb{N}^+$ with $x^n + y > 0$.