Written Assignment 7: Due Wednesday, November 9

Problem 1: In our definition of the elementary row operation known as "row combination", we replace row i by the sum of itself and a multiple of a different row j (where different means that $j \neq i$). Suppose then that we consider the operation where we take a row i and replace it by the sum of itself and a multiple of row i. Do we necessarily preserve the solution set of the system by doing this? As always, you must explain if your answer is yes, or you must provide a specific counterexample (with justification) if your answer is no.

Problem 2: Show that for all $a, b, c \in \mathbb{R}$, the matrices

$$\begin{pmatrix} 4 & 2 & 1 \\ a & -1 & 0 \\ b & c & 3 \end{pmatrix}$$

and

$$\begin{pmatrix} 1 & 1 & 2 \\ -2 & 0 & -1 \\ 1 & 3 & 5 \end{pmatrix}$$

are not row equivalent, i.e. there does not exist a sequence of elementary row operations that turns the first matrix into the second matrix.

Problem 3: Let V be a vector space. Suppose that U and W are both subspaces of V. We showed in Written Assignment 6 that $U \cup W$ might not be a subspace of V. Instead, let

$$U + W = \{\vec{v} \in V : \text{There exists } \vec{u} \in U \text{ and } \vec{w} \in W \text{ with } \vec{v} = \vec{u} + \vec{w}\}.$$

That is, $U + W$ is the set of all vectors in V that can be written as the sum of an element of U and an element of W. Show that $U + W$ is a subspace of V.