Written Assignment 6: Due Wednesday, November 2

Recall: When we use the word *or* in mathematics, we mean that at least one of the options happens (so this is the "inclusive" or). In other words, when we say "A or B", we are allowing the possibility that both A and B are true. Be sure to interpret *or* in this way in both Problem 1a and Problem 2b.

Problem 1: Let V be a vector space. Prove each of the following. For this problem, be very explicit and mention which property and/or result you are using in each step of your argument.
a. Suppose that $c \in \mathbb{R}$ and $\vec{v} \in V$ are such that $c \cdot \vec{v} = \vec{0}$. Show that either $c = 0$ or $\vec{v} = \vec{0}$.
b. Suppose that $c, d \in \mathbb{R}$ and $\vec{v} \in V$ are such that $c \cdot \vec{v} = d \cdot \vec{v}$. Show that if $\vec{v} \neq \vec{0}$, then $c = d$.
c. Show that if a vector space has more than 1 element, then it must have infinitely many elements.
Hint for a: In mathematics, one of the standard ways to prove a statement of the form "A or B" is to assume that A is false, and use this to prove that B must be true (or alternatively to assume that B is false, and use this to prove that A must be true). This allows you to use an additional assumption, which is extremely useful. In this case, I suggest that you also assume that $c \neq 0$ (in addition to $c \cdot \vec{v} = \vec{0}$), and then prove that $\vec{v} = \vec{0}$.

Problem 2: Let V be a vector space. Suppose that U and W are both subspaces of V.
a. Let $U \cap W$ be the intersection of U and W, i.e. $U \cap W = \{\vec{v} \in V : \vec{v} \in U \text{ and } \vec{v} \in W\}$. Show that $U \cap W$ is a subspace of V.
b. Let $U \cup W$ be the union of U and W, i.e. $U \cup W = \{\vec{v} \in V : \vec{v} \in U \text{ or } \vec{v} \in W\}$. By constructing an explicit example, show that $U \cup W$ need *not* be a subspace of V.