Problem Set 3: Due Friday, September 9

Problem 1: Write both the converse and contrapositive of each of the following statements (no need to argue whether any of the them are true or false). In each case, get rid of all occurrences of *not* in the final result.
a. If $a \in \mathbb{Z}$ and $a \geq 2$, then $4a > 7$.
b. If $x, y \in \mathbb{R}$ and $x^4 + y^4 = 1$, then $x^2 + y^2 \leq 2$.
c. If $a \in \mathbb{Z}$ and there exists $m \in \mathbb{Z}$ with $a = 10m$, then there exists $m \in \mathbb{Z}$ with $a = 5m$.

Problem 2: Consider the following statement:

If $a \in \mathbb{Z}$ and $3a + 5$ is even, then a is odd.

a. Write down the contrapositive of the given statement.
b. Show that the original statement is true by proving that the contrapositive is true.

Problem 3: Let $A = \{e^x : x \in \mathbb{R}\}$.
a. Write a description of A by carving it out of a set using a property with a "there exists" quantifier.
b. Find another way to describe A by carving it out of a set using a property without any quantifiers. Briefly explain why your set is equal (no need to give a formal proof).

Problem 4: Let $A = \{12n - 7 : n \in \mathbb{Z}\}$ and let $B = \{4n + 1 : n \in \mathbb{Z}\}$.
a. Show that $B \not\subseteq A$.
b. Fill in the blanks below with appropriate phrases so that the result is a correct proof of the statement that $A \subseteq B$.

Let $a \in A$ be arbitrary. By definition of A, we can ________________. Now notice that $a =$ ________________. Since ________________ $\in \mathbb{Z}$, we conclude that $a \in B$. Since $a \in A$ was arbitrary, the result follows.

Problem 5: Let $A = \{x^2 + 5 : x \in \mathbb{R}\}$ and let $B = \{x \in \mathbb{R} : x \geq 5\}$. In this problem, we show that $A = B$ by doing a double containment proof.
a. Prove that $A \subseteq B$.
b. Fill in the blanks below with appropriate phrases so that the result is a correct proof of the statement that $B \subseteq A$.

Let $y \in B$ be arbitrary. By definition of B, we know ________________. Now notice that ________________ ≥ 0 so ________________ $\in \mathbb{R}$, and that ________________ $= y$, so $y \in A$. Since $y \in B$ was arbitrary, the result follows.