Problem Set 23: Due Friday, December 9

Problem 1: Calculate

$$\begin{vmatrix} 3 & 0 & -2 & 1 \\ 4 & 0 & 2 & 0 \\ -5 & 2 & -8 & 7 \\ 3 & 0 & 3 & -1 \end{vmatrix}.$$

Problem 2: Suppose that

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 5.$$

Find, with explanation, the value of

$$\begin{vmatrix} -d & -e & -f \\ 2g+3a & 2h+3b & 2i+3c \\ a & b & c \end{vmatrix}.$$

Problem 3: Show that

$$\begin{vmatrix} a & b & c \\ a+x & b+x & c+x \\ a+y & b+y & c+y \end{vmatrix} = 0$$

for all $a, b, c, x, y \in \mathbb{R}$.

Problem 4: Given $c \in \mathbb{R}$, consider the matrix

$$A_c = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 9 & c \\ 1 & c & 3 \end{pmatrix}.$$

a. Use a cofactor expansion to compute $\det(A_c)$.
b. Find all values of c such that A_c is invertible. Explain.

Problem 5: Find a basis for the eigenspace of the matrix

$$\begin{pmatrix} 1 & 4 & 1 \\ 6 & 6 & 2 \\ -3 & -4 & -3 \end{pmatrix}$$

corresponding to $\lambda = -2$.