Problem Set 19: Due Friday, November 18

Problem 1: Let

$$\alpha = \left(\begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}, \begin{pmatrix} -1 \\ -3 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 8 \\ 9 \end{pmatrix} \right).$$

a. Show that α is a basis of $\mathbb{R}^3$.
b. Determine

$$\left[\begin{pmatrix} 1 \\ 5 \\ -5 \end{pmatrix} \right]_\alpha.$$

Problem 2: Consider the following elements of $\mathcal{P}_3$:

- $f_1(x) = x^3$.
- $f_2(x) = x^3 + x^2$.
- $f_3(x) = x^3 + x^2 + x$.
- $f_4(x) = x^3 + x^2 + x + 1$.

Let $\alpha = (f_1, f_2, f_3, f_4)$.
a. Show that α is a basis of $\mathcal{P}_3$.
b. Let $g(x) = 3x^3 + 7x^2 + 7x - 2$. Determine $[g]_\alpha$.

Problem 3: Let $W = \{f \in \mathcal{P}_2 : f(2) = 0\}$. It can be checked that W is a subspace of $\mathcal{P}_2$ (no need to do this). Let $\alpha = (f_1, f_2)$ where:

- $f_1(x) = x^2 - 4$.
- $f_2(x) = x - 2$.

a. Show that α is a basis of W, and determine $\dim(W)$.
b. Let $g(x) = 2x^2 - 7x + 6$. Determine $[g]_\alpha$.

Problem 4: Let V be the vector space of all 2×2 matrices. Let

$$W = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in V : b = c \right\}.$$

It can be checked that W is a subspace of V (no need to do this). Find a basis for W, and determine $\dim(W)$.

Problem 5: In Problem 2 on Problem Set 18, you showed that

$$\left(\begin{pmatrix} 0 \\ 1 \\ 3 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 2 \\ -1 \end{pmatrix}, \begin{pmatrix} -8 \\ 2 \\ -2 \\ 2 \end{pmatrix}, \begin{pmatrix} 6 \\ -1 \\ 9 \\ 5 \end{pmatrix} \right)$$

was linearly dependent. Use your work in that problem to find a basis (with explanation) for the following subspace of $\mathbb{R}^4$:

$$W = \text{Span} \left(\begin{pmatrix} 0 \\ 1 \\ 3 \\ -1 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 2 \\ -1 \end{pmatrix}, \begin{pmatrix} -8 \\ 2 \\ -2 \\ 2 \end{pmatrix}, \begin{pmatrix} 6 \\ -1 \\ 9 \\ 5 \end{pmatrix} \right).$$