Problem Set 16: Due Monday, November 7

Problem 1: Use Gaussian Elimination to solve the following system:

$$\begin{array}{rcrcrcl} x & & & - & z & = & 0 \\ 3x & + & y & & & = & 1 \\ -x & + & y & + & z & = & 4. \end{array}$$

Problem 2: Find the coefficients a, b, and c so that the graph of $f(x) = ax^2 + bx + c$ passes through the points $(1, 2)$, $(-1, 6)$, and $(2, 3)$.

Problem 3: Is

$$\begin{pmatrix} 20 \\ 0 \\ 5 \\ 10 \end{pmatrix} \in \text{Span}\left(\begin{pmatrix} 0 \\ 2 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 4 \\ -2 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \\ -1 \end{pmatrix} \right)?$$

Explain.

Problem 4: Give a parametric description of the solution set of the following system:

$$\begin{array}{rcrcrcrcl} x & + & 2y & - & z & & & = & 3 \\ 2x & + & y & & & + & w & = & 4 \\ x & - & y & + & z & + & w & = & 1. \end{array}$$

Problem 5: Use Gaussian Elimination to determine for which values of $h, k \in \mathbb{R}$ the system

$$\begin{array}{rcrcl} x & + & hy & = & 2 \\ 4x & + & 8y & = & k \end{array}$$

has each of the following: (i) no solution, (ii) one solution, and (iii) infinitely many solutions.

Problem 6: Determine exact conditions (that is, conditions that are both necessary and sufficient) on $a, b, c, d \in \mathbb{R}$ such that

$$\begin{array}{rcrcl} x & - & 3y & = & a \\ 3x & + & y & = & b \\ x & + & 7y & = & c \\ 2x & + & 4y & = & d \end{array}$$

has a solution.