Problem Set 1: Due Friday, September 2

Problem 1: Let P be the plane in $\mathbb{R}^3$ containing the origin and both of the following two vectors:

$$\vec{u} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} \quad \text{and} \quad \vec{w} = \begin{pmatrix} -7 \\ 1 \\ 4 \end{pmatrix}$$

In the notes, we discussed one way to parametrize P, and we will discuss this in more detail later. Now find an equation of the form $ax + by + cz = d$ for P. Explain your process using a sentence of two.

Problem 2: Let L be the line in $\mathbb{R}^3$ that is the intersection of the two planes $3x + 4y - z = 2$ and $x - 2y + z = 4$.

a. Using the equations of the planes, determine if the points $(1, 0, 1)$ and $(1, 1, 5)$ are on L.
b. Find a parametric description of L. Explain your process using a sentence or two.
c. Use the parametric description of L to determine if $(5, 2, 3)$ is a point on L. Explain.
Note: Given a point, it seems easier to determine if it is on L using the equations of the planes rather than the parametric description. In contrast, if you want to *generate* points on L, it is easier to use the parametric description (just plug in values for the parameter) than the plane equations.

Problem 3:

a. Do the planes with equations $2x - 3y + z = 7$ and $-4x + 9y - 2z = 3$ intersect? Explain your reasoning.
b. Do the lines described by the two parametric equations

$$\begin{aligned} x &= -4 + 6t \\ y &= 2 + t \\ z &= 1 + 3t \end{aligned} \qquad\qquad \begin{aligned} x &= 4 + 4t \\ y &= 5 - t \\ z &= 9 - 2t \end{aligned}$$

intersect? Explain your reasoning.

Problem 4: Determine if the following are true or false. Justify your answers using complete sentences in each case.

a. There exists $x \in \mathbb{R}$ with $\sin x = \cos x$.
b. There exists $x \in \mathbb{R}$ with $\sin x = \cos x + 2$.
c. There exists $m, n \in \mathbb{N}$ with $9m + 15n = 3$.
d. There exists $m, n \in \mathbb{Z}$ with $9m + 15n = 3$.
e. For all $t \in \mathbb{R}$, we have

$$2\cos^4(3t) + 2\cos^2(3t) \cdot \sin^2(3t) - \cos(6t) = 1.$$

f. For all $a \in \mathbb{R}$, we have $a^2 + 6a + 10 > 0$. (Do not rely on a picture.)