

## Written Assignment 7: Due Wednesday, November 12

**Problem 1:** In our definition of the elementary row operation known as “row combination”, we replace row  $i$  by the sum of itself and a multiple of a different row  $j$  (where different means that  $j \neq i$ ). Suppose then that we consider the operation where we take a row  $i$  and replace it by the sum of itself and a multiple of row  $i$ . Do we necessarily preserve the solution set of the system by doing this? As always, you must explain if your answer is yes, or you must provide a specific counterexample (with justification) if your answer is no.

**Problem 2:** Show that for all  $a, b, c \in \mathbb{R}$ , there does not exist a sequence of elementary row operations that turns

$$\begin{pmatrix} 4 & 2 & 1 \\ a & -1 & 0 \\ b & c & 3 \end{pmatrix}$$

into

$$\begin{pmatrix} 1 & 1 & 2 \\ -2 & 0 & -1 \\ 1 & 3 & 5 \end{pmatrix}$$

**Problem 3:** Let  $V$  be a vector space. Suppose that  $U$  and  $W$  are both subspaces of  $V$ . We showed in Written Assignment 6 that  $U \cup W$  might not be a subspace of  $V$ . Instead, let

$$U + W = \{\vec{v} \in V : \text{There exists } \vec{u} \in U \text{ and } \vec{w} \in W \text{ with } \vec{v} = \vec{u} + \vec{w}\}.$$

That is,  $U + W$  is the set of all vectors in  $V$  that can be written as the sum of an element of  $U$  and an element of  $W$ . Show that  $U + W$  is a subspace of  $V$ .