

Written Assignment 6: Due Wednesday, November 5

Recall: When we use the word *or* in mathematics, we mean that at least one of the options happens (so this is the “inclusive” or). In other words, when we say “ A or B ”, we are allowing the possibility that both A and B are true. Be sure to interpret *or* in this way in both Problem 1a and Problem 2b.

Problem 1: Let V be a vector space. Prove each of the following. For this problem, be very explicit and mention which property and/or result you are using in each step of your argument.

a. Suppose that $c \in \mathbb{R}$ and $\vec{v} \in V$ are such that $c \cdot \vec{v} = \vec{0}$. Show that either $c = 0$ or $\vec{v} = \vec{0}$.

b. Suppose that $\vec{v} \in V$ and $c, d \in \mathbb{R}$ are such that $c \cdot \vec{v} = d \cdot \vec{v}$. Show that if $\vec{v} \neq \vec{0}$, then $c = d$.

c. Show that if a vector space has more than 1 element, then it must have infinitely many elements.

Hint for a: In mathematics, one of the standard ways to prove a statement of the form “ A or B ” is to assume that A is false, and use this to prove that B must be true (or alternatively to assume that B is false, and use this to prove that A must be true). This allows you to use an additional assumption, which is extremely useful.

Problem 2: Let V be a vector space. Suppose that U and W are both subspaces of V .

a. Let $U \cap W$ be the intersection of U and W , i.e. $U \cap W = \{\vec{v} \in V : \vec{v} \in U \text{ and } \vec{v} \in W\}$. Show that $U \cap W$ is a subspace of V .

b. Let $U \cup W$ be the union of U and W , i.e. $U \cup W = \{\vec{v} \in V : \vec{v} \in U \text{ or } \vec{v} \in W\}$. By constructing an explicit example, show that $U \cup W$ need *not* be a subspace of V .