# Problem Set 23: Due Friday, December 12

**Problem 1:** Calculate

$$\begin{vmatrix} 3 & 0 & -2 & 1 \\ 4 & 0 & 2 & 0 \\ -5 & 2 & -8 & 7 \\ 3 & 0 & 3 & -1 \end{vmatrix}$$

**Problem 2:** Suppose that

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 5$$

Find, with explanation, the value of

$$\begin{vmatrix} -d & -e & -f \\ 2g+3a & 2h+3b & 2i+3c \\ a & b & c \end{vmatrix}$$

**Problem 3:** Show that

$$\begin{vmatrix} a & b & c \\ a+x & b+x & c+x \\ a+y & b+y & c+y \end{vmatrix} = 0$$

for all $a, b, c, x, y \in \mathbb{R}$.

**Problem 4:** Given $c \in \mathbb{R}$, consider the matrix

$$A_c = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 9 & c \\ 1 & c & 3 \end{pmatrix}$$

a. Use a cofactor expansion to compute $\det(A_c)$.
b. Find all values of $c$ such that $A_c$ is invertible. Explain.

**Problem 5:** Find a basis for the eigenspace of the matrix

$$\begin{pmatrix} 1 & 4 & 1 \\ 6 & 6 & 2 \\ -3 & -4 & -3 \end{pmatrix}$$

corresponding to $\lambda = -2$.