Problem Set 22: Due Monday, December 8

Problem 1: Consider the matrix

$$A = \begin{pmatrix} 3 & 2 \\ 5 & 3 \end{pmatrix}$$

We know from Proposition 2.55 that A is invertible, and we also know a formula for the inverse. Now compute A^{-1} using our new method by applying elementary row operations to the matrix

$$\begin{pmatrix} 3 & 2 & 1 & 0 \\ 5 & 3 & 0 & 1 \end{pmatrix}$$

Problem 2: Let V be the vector space of all 2×2 matrices. Explain why there is no injective linear transformation $T \colon \mathcal{P}_4 \to V$.

Problem 3: Determine whether each of the following matrices is invertible, and if so, find the inverse.

a. $\begin{pmatrix} 1 & 1 & 3 \\ 0 & 2 & 4 \\ -1 & 1 & 0 \end{pmatrix}$

b. $\begin{pmatrix} 0 & 4 & 4 \\ 1 & -2 & 0 \\ 3 & -4 & 2 \end{pmatrix}$

c. $\begin{pmatrix} 0 & 1 & 5 \\ 0 & -2 & 4 \\ 2 & 3 & -2 \end{pmatrix}$

Problem 4: Either prove or find a counterexample: If A and B are invertible $n \times n$ matrices, then $A + B$ is invertible.

Problem 5: Consider the matrix

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

a. Explain why A has no left inverse.

b. Show that A has infinitely many right inverses.