Problem Set 20: Due Monday, November 24

Problem 1: Working in $\mathbb{R}^4$, let

$$W = Span\left(\begin{pmatrix}0\\0\\1\\3\end{pmatrix}, \begin{pmatrix}4\\5\\2\\7\end{pmatrix}, \begin{pmatrix}7\\8\\0\\1\end{pmatrix}\right)$$

Explain why $\dim(W) = 3$.

Problem 2: Let V be the vector space of all 2×2 matrices. Let

$$W = \left\{\begin{pmatrix}a & b\\c & d\end{pmatrix} \in V : 2a - c = 0 \text{ and } b + c - d = 0\right\}$$

It turns out that W is a subspace for V (no need to show this). Find a basis for W, and determine $\dim(W)$.
Hint: First try to write W as the span of some elements of V by solving the system of equations.

Problem 3: Define $T\colon \mathcal{P}_1 \to \mathbb{R}^2$ by letting

$$T(a + bx) = \begin{pmatrix}a - b\\b\end{pmatrix}$$

Show that T is a linear transformation.

Problem 4: Let $T\colon \mathbb{R}^3 \to \mathbb{R}^3$ be the function

$$T\left(\begin{pmatrix}x\\y\\z\end{pmatrix}\right) = \begin{pmatrix}x - y\\x + z\\y + z\end{pmatrix}$$

a. Explain why T is a linear transformation.
b. Give an example of a nonzero $\vec{v} \in \mathbb{R}^3$ such that $T(\vec{v}) = \vec{0}$.
c. Show that T is not injective.

Problem 5: Let V be the vector space of all 2×2 matrices. Define $T\colon V \to \mathbb{R}$ by letting

$$T\left(\begin{pmatrix}a & b\\c & d\end{pmatrix}\right) = 2a - d.$$

a. Show that T is a linear transformation.
b. Show that T is surjective.