

Problem Set 17: Due Friday, November 14

Problem 1: Does

$$\text{Span} \left(\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right) = \mathbb{R}^3?$$

Explain.

Problem 2: Given $b_1, b_2, b_3 \in \mathbb{R}$, determine necessary and sufficient conditions so that

$$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \in \text{Span} \left(\begin{pmatrix} 0 \\ 1 \\ 5 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix} \right)$$

is true.

Problem 3: Working in \mathcal{P}_3 , consider the following functions:

- $f_1(x) = x^3 + 2x^2 + x$.
- $f_2(x) = -3x^3 - 5x^2 + x + 2$.
- $f_3(x) = x^2 - x + 1$.
- $g(x) = x^3 + 8x^2 + 7$.

Is $g \in \text{Span}(f_1, f_2, f_3)$? Explain.

Problem 4: Let V be the vector space of all 2×2 matrices. Does

$$\text{Span} \left(\begin{pmatrix} 1 & 1 \\ 2 & 0 \end{pmatrix}, \begin{pmatrix} 2 & 3 \\ 7 & 2 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 2 & 6 \end{pmatrix} \right) = V?$$

Explain.

Problem 5: Show that the only subspaces of \mathbb{R} are $\{0\}$ and \mathbb{R} .

Hint: Suppose that W is a subspace of \mathbb{R} with $W \neq \{0\}$. Explain why every element of \mathbb{R} is in W .

Problem 6: In Problem 5 on Problem Set 14, you showed that

$$W = \left\{ \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \in \mathbb{R}^3 : a_1 + a_2 + a_3 = 0 \right\}$$

was a subspace of \mathbb{R}^3 . Show that

$$W = \text{Span} \left(\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \right)$$

by giving a double containment proof.

Aside: Using this result, we can instead apply Proposition 3.16 to conclude that W is a subspace of \mathbb{R}^3 .