Problem Set 13: Due Wednesday, October 29

Problem 1: Consider the unique linear transformation $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ with

$$[T] = \begin{pmatrix} 1 & 0 \\ 6 & -1 \end{pmatrix}$$

Determine if T is diagonalizable. If so, find an example of $\alpha = (\vec{u}_1, \vec{u}_2)$ with $Span(\vec{u}_1, \vec{u}_2) = \mathbb{R}^2$ such that $[T]_\alpha$ is a diagonal matrix, and determine $[T]_\alpha$ in this case.

Problem 2: Consider the unique linear transformation $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ with

$$[T] = \begin{pmatrix} 3 & -1 \\ 1 & 5 \end{pmatrix}$$

Determine if T is diagonalizable. If so, find an example of $\alpha = (\vec{u}_1, \vec{u}_2)$ with $Span(\vec{u}_1, \vec{u}_2) = \mathbb{R}^2$ such that $[T]_\alpha$ is a diagonal matrix, and determine $[T]_\alpha$ in this case.

Problem 3: Define a sequence of numbers as follows. Let $g_0 = 0$, $g_1 = 1$, and $g_n = \frac{1}{2}(g_{n-1} + g_{n-2})$ for all $n \in \mathbb{N}$ with $n \geq 2$. In other words, if $n \geq 2$, then the n^{th} term of the sequence is the average of the two previous terms.
a. Write down a 2×2 matrix A such that

$$A \begin{pmatrix} g_{n+1} \\ g_n \end{pmatrix} = \begin{pmatrix} g_{n+2} \\ g_{n+1} \end{pmatrix}$$

for all $n \in \mathbb{N}$.
b. Find an invertible matrix P and a diagonal matrix D with $A = PDP^{-1}$.
c. Find a general equation for g_n.
d. As n gets large, the values of g_n approach a fixed number. Find that number.

Problem 4: Given a 2×2 matrix A and an $r \in \mathbb{R}$, what is the relationship between $\det(r \cdot A)$ and $\det(A)$? Explain.