Written Assignment 9: Due Wednesday, December 11

Required Problems

Problem 1: Suppose that A is an $n \times n$ idempotent matrix (recall this means that $A^2 = A$).
a. Show that the only possible eigenvalues of A are 0 and 1.
b. Suppose also that $A \neq I$. Show that 0 is in fact an eigenvalue of A.

Problem 2: Let P be an $n \times n$ matrix such that

- All entries in P are nonnegative.
- Every column sums to 1.

Such a matrix is called a *stochastic matrix*. In this problem we prove that there exists a nonzero vector $\vec{v} \in \mathbb{R}^n$ with $P\vec{v} = \vec{v}$ (i.e. that 1 is an eigenvalue of P).
a. Show that if you add up the rows of $P - I$, you get the zero vector.
b. Show that $\text{rank}(P - I) < n$.
c. Show that there exists a nonzero vector $\vec{v} \in \mathbb{R}^n$ with $P\vec{v} = \vec{v}$.

Challenge Problems

Problem 1: Let A be an $n \times n$ matrix and suppose that every vector in $\mathbb{R}^n$ is an eigenvector of A (but do not assume that they all correspond to the same eigenvalue). Show that there exists a scalar c such that $A = cI$.