Written Assignment 6: Due Wednesday, November 6

Required Problems

Problem 1: Let V and W be vector spaces. Suppose that $t\colon V \to W$ is a surjective linear transformation and that $V = [\vec{u}_1, \dots, \vec{u}_n]$. In this problem, you will show that $W = [t(\vec{u}_1), \dots, t(\vec{u}_n)]$.
a. To show that $W = [t(\vec{u}_1), \dots, t(\vec{u}_n)]$, you need to start by taking an arbitrary element of which set? Using that element, what is your goal?
b. Starting with your element in part a, use the fact that t is surjective to obtain a new element.
c. Use the fact that $V = [\vec{u}_1, \dots, \vec{u}_n]$.
d. Use the fact that t is a linear transformation to reach your goal.

Problem 2: Let V and W be vector spaces. Suppose that $t\colon V \to W$ is an injective linear transformation and that $\{\vec{u}_1, \dots, \vec{u}_n\}$ is a linearly independent subset of V. In this problem, you will show that $\{t(\vec{u}_1), \dots, t(\vec{u}_n)\}$ is a linearly independent subset of W.
a. To show that $\{t(\vec{u}_1), \dots, t(\vec{u}_n)\}$ is a linearly independent set, you need to assume that a certain equation is true and deduce something. Carefully and formally write down this assumption and your goal.
b. Starting with your assumed equation in part a, use the fact that that t is a linear transformation to derive a new equation.
c. Use the assumption that t is injective.
d. Use the fact that $\{\vec{u}_1, \dots, \vec{u}_n\}$ is linearly independent to reach your goal.

Challenge Problems

Problem 1: Give an example, with justification, of a vector space V and a linear transformation $t\colon V \to V$ such that t is injective but not surjective.

Problem 2: Let $\vec{w} \in \mathbb{R}^2$ be a nonzero vector and let $W = [\vec{w}]$. Define a function $t\colon \mathbb{R}^2 \to W$ by letting $t(\vec{v})$ be the point in W that is closest to $\vec{v}$ (i.e. $t(\vec{v})$ is the projection of the vector $\vec{v}$ onto the line W). Derive a formula for t based on the coordinates of $\vec{w}$, and prove that t is a linear transformation.